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LETTER TO THE EDITOR

Self-attracting walk with $\nu < 1/2$

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Abstract. A model of a self-attracting walk is proposed. Mean square displacement of a particle ξ grows with time t as a power law $\xi^2 \sim t^{2\nu}$ with the exponent $\nu < 1/2$. It is shown that $\nu = 1/(2D - D_b)$ where D is the fractal dimension of the cluster consisting of the visited sites and D_b is the fractal dimension of its boundary. A compact cluster with fractal boundaries was obtained by a computer simulation for $d = 2$. The derived value of ν is in a good agreement with that obtained by computer simulation.

The models of a random walk with memory were found to be an effective instrument for the investigation of those characteristics of various physical processes which are responsible for their fractal properties. In particular, it was shown that the trajectory of the SAW simulates the linear polymers [1] and its statistical properties correspond to those of the phase transition in the model of a ferromagnetic [2]. For the models of a random walk with repulsion, such as SAW and TSAW [3], mean square displacement of a particle from its initial position ξ grows with time t as a power law $\xi^2 \sim t^{2\nu}$. An essential feature of these processes is their persistence expressed by an inequality $\nu > \nu_{\text{brown}} = 1/2$ (ν_{brown} is an exponent for a simple random walk) which means positive correlation of the previous and the subsequent walks.

However, up to the present time no models have been built which would be characterized by anti-persistence ($\nu < 1/2$). Creation and study of such models is a high-priority task because many phenomena are revealed such as the evolution of a surface of growing aggregates (see, for example, references in [4]) which show exponents less than 1/2. In this work a simple model of a self-attracting walk (SATW) is proposed which shows anti-persistence. A model of a self-attracting walk has been studied in one dimension in [5]. However, the authors did not find the exponent ν. Let the probability for a particle to jump to a given site be $p \sim \exp(-nu)$, where $n = 1$ for the sites visited by the particle at least once ('black sites') and $n = 0$ for the others ('white sites'). If $u < 0$ the particle attracts to its own trajectory. As we will see, for such a walk $\nu < 1/2$. Let first the value u be arbitrary, including zero. Let us consider the cluster consisting of the visited sites (s black sites). Let the boundary length of the cluster, i.e. the number of the black sites which have white neighbours be l_b. Considering that the particle can be located at any black site with equal probability we come to the conclusion that the rate of the cluster growth is proportional to the fraction of the boundary sites in respect to the total number of the black sites. Indeed, the cluster has a chance to increase only at the moment when the particle is located on the boundary. Thus,

$$\frac{\mathrm{d}s}{\mathrm{d}t} \sim \frac{l_b}{s}. \tag{1}$$

Considering that the fractal dimension of the cluster is D and that of its boundary is D_b we can rewrite (1) as

$$\frac{\mathrm{d}R^D}{\mathrm{d}t} \sim \frac{R^{D_b}}{R^D} \qquad (2)$$

where R is the characteristic geometrical size of the cluster. Hence we get

$$R \sim t^{1/(2D-D_b)}. \qquad (3)$$

Taking into account that $\xi^2 \sim R^2$ we get finally

$$\nu = 1/(2D - D_b). \qquad (4)$$

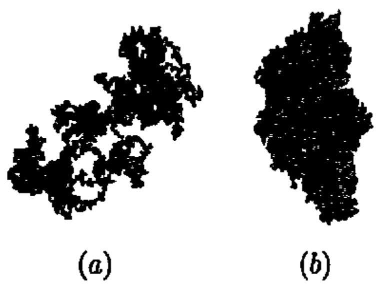

Figure 1. Clusters formed by a particle walk: (*a*) $u = 0$, 3×10^4 jumps, (*b*) $u = -2$, 10^6 jumps.

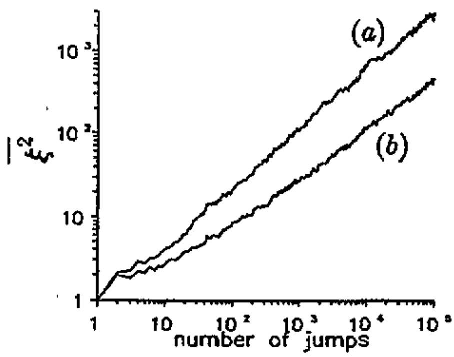

Figure 2. Mean square displacement of the particle from its initial position: (*a*) $u = -1$, averaged over 200 runs, (*b*) $u = -2$, averaged over 500 runs.

For a one-dimensional case $D = d = 1$, $D_b = 0$ and the relationship (4) gives $\nu = 1/2$ regardless of the u value. For $d = 2$ I carried out a computer simulation on a square lattice with 300×300 sites at $u = 0$, $u = -1$ and $u = -2$. The fractal dimensions were found by counting the number of the cluster or the boundary sites within a box. At $u = 0$ (figure 1(*a*)) the fractal dimension of the cluster D and the dimension of its boundary D_b wre close to 1.75 and 1.5, respectively. In this case the relationship (4) gives the well known value for a simple random walk $\nu = 0.5$. One should not be surprised by the fact that $D < 2$ because it characterizes the scaling properties of the black cluster, not of the trajectory of the particle, so each black site is taken into account only once, regardless of the number of visits. A trivial example is $D = d = 1$ (i.e. $D < 2$) for one-dimensional walk.

At $u = -1$ and $u = -2$ $D = d = 2$, i.e. the cluster is compact (figure 1(*b*)). It has a fractal boundary and some pores localized in the vicinity of the boundary. The fractal dimension of the boundary, including the boundary of the pores, was $D_b = 1.4$ for $u = -1$ and $D_b = 1.2$ for $u = -2$. Hence, the relationship (4) gives $\nu = 0.38$ for $u = -1$ and $\nu = 0.36$ for $u = -2$. Computer simulation (figure 2) gives the values $\nu = 0.38$ and $\nu = 0.32$, respectively, in a satisfactory agreement with the analytical results. The lower value of ν for $u = -2$ may be due to the fact that it has not reached its asymptotic value after 10^5 particle jumps.

It seems that there is a critical value u_c such that $\nu < 1/2$ only for $u < u_c$. Thus, the simulation for $u = -0.5$ gives $\nu = 0.49 \approx 1/2$ both by direct measurement and from the relationship (4).

Supposing that for the $d = 3$ case the cluster is also compact for $u < u_c$ and taking into account that in this case $2 < D_b < 3$ we find from (4) that $1/4 < \nu < 1/3$.

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